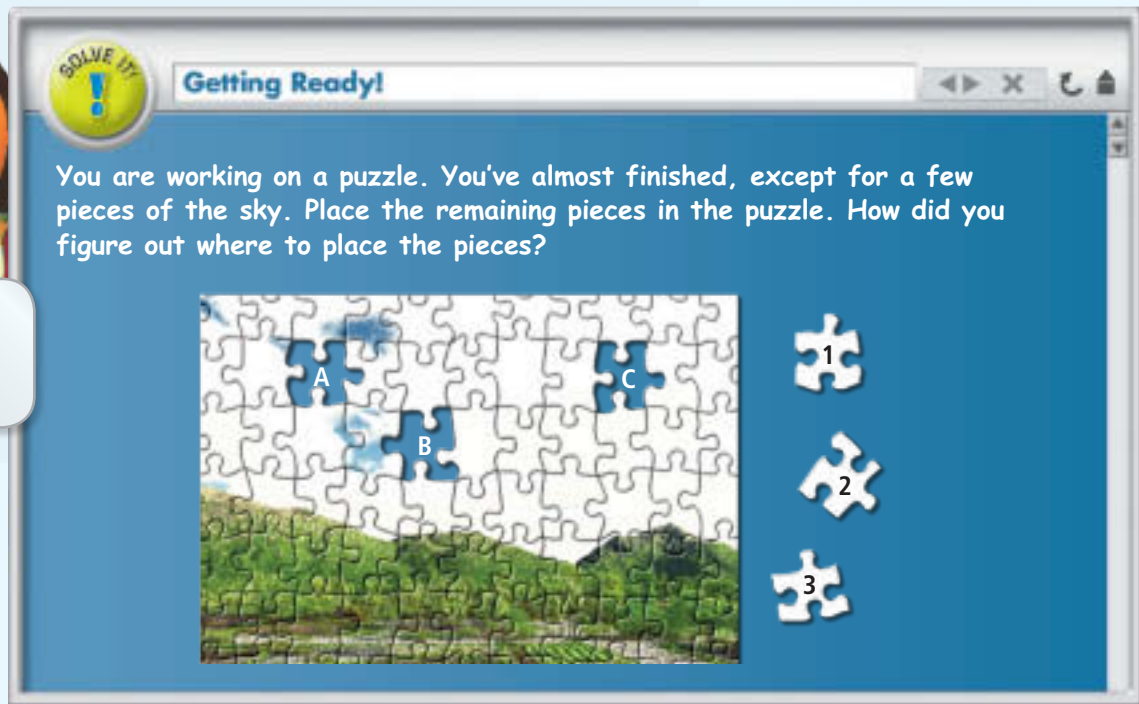


**Objective** To recognize congruent figures and their corresponding parts



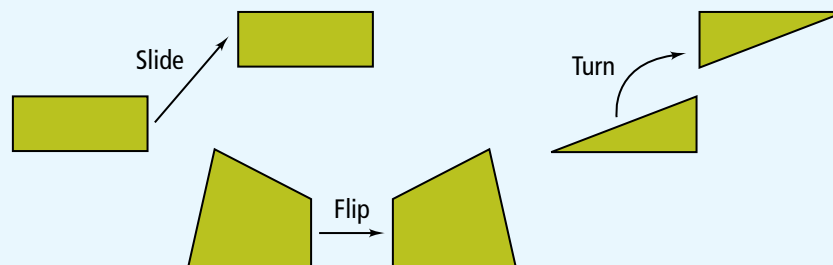
Having trouble?  
How can tracing  
pieces 1, 2, and 3  
help?



**Lesson Vocabulary**

- congruent polygons

Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one, as shown below. In this lesson, you will learn how to determine if geometric figures are congruent.



**Essential Understanding** You can determine whether two figures are congruent by comparing their corresponding parts.

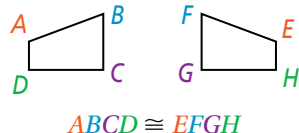
Take note

## Key Concept Congruent Figures

### Definition

**Congruent polygons** have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.

### Example



$$\begin{array}{ll} \overline{AB} \cong \overline{EF} & \overline{BC} \cong \overline{FG} \\ \overline{CD} \cong \overline{GH} & \overline{DA} \cong \overline{HE} \\ \angle A \cong \angle E & \angle B \cong \angle F \\ \angle C \cong \angle G & \angle D \cong \angle H \end{array}$$

### Plan

How do you know which sides and angles correspond? The congruence statement  $HJK \cong LMNO$  tells you which parts correspond.

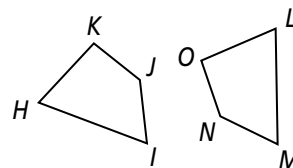


### Problem 1 Finding Congruent Parts

If  $HJK \cong LMNO$ , what are the congruent corresponding parts?

Sides:  $\overline{HI} \cong \overline{LM}$   $\overline{IJ} \cong \overline{MN}$   $\overline{JK} \cong \overline{NO}$   $\overline{KH} \cong \overline{OL}$

Angles:  $\angle H \cong \angle L$   $\angle I \cong \angle M$   $\angle J \cong \angle N$   $\angle K \cong \angle O$



**Got It?** 1. If  $\triangle WYS \cong \triangle MKV$ , what are the congruent corresponding parts?

### Plan

You know two angle measures in  $\triangle ABC$ . How can they help? In the congruent triangles,  $\angle D$  corresponds to  $\angle A$ , so you know that  $\angle D \cong \angle A$ . You can find  $m\angle D$  by first finding  $m\angle A$ .



### Problem 2 Using Congruent Parts

**Multiple Choice** The wings of an SR-71 Blackbird aircraft suggest congruent triangles. What is  $m\angle D$ ?

- (A) 30 (B) 75 (C) 105 (D) 150

### Think

Use the Triangle Angle-Sum Theorem to write an equation involving  $m\angle A$ .

Solve for  $m\angle A$ .

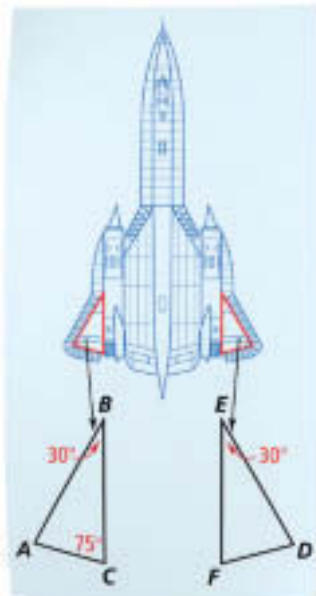
$\angle A$  and  $\angle D$  are corresponding parts of congruent triangles, so  $\angle A \cong \angle D$ .

### Write

$$m\angle A + 30 + 75 = 180$$

$$\begin{array}{l} m\angle A + 105 = 180 \\ m\angle A = 75 \end{array}$$

$$\begin{array}{l} m\angle A = m\angle D = 75 \\ \text{The correct answer} \\ \text{is B.} \end{array}$$



**Got It?** 2. Suppose that  $\triangle WYS \cong \triangle MKV$ . If  $m\angle W = 62$  and  $m\angle Y = 35$ , what is  $m\angle V$ ? Explain.

## Plan

How do you determine whether two triangles are congruent?

Compare each pair of corresponding parts. If all six pairs are congruent, then the triangles are congruent.



### Problem 3 Finding Congruent Triangles

Are the triangles congruent? Justify your answer.

$$\overline{AB} \cong \overline{ED}$$

Given

$$\overline{BC} \cong \overline{DC}$$

$$BC = 4 = DC$$

$$\overline{AC} \cong \overline{EC}$$

$$AC = 6 = EC$$

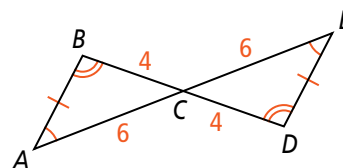
$$\angle A \cong \angle E, \angle B \cong \angle D$$

Given

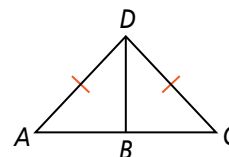
$$\angle BCA \cong \angle DCE$$

Vertical angles are congruent.

$\triangle ABC \cong \triangle EDC$  by the definition of congruent triangles.



**Got It?** 3. Is  $\triangle ABD \cong \triangle CBD$ ? Justify your answer.



Recall the Triangle Angle-Sum Theorem: The sum of the measures of the angles in a triangle is 180. The next theorem follows from the Triangle Angle-Sum Theorem.

take note

### Theorem 4-1 Third Angles Theorem

#### Theorem

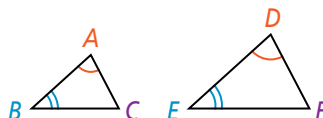
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

If ...

$$\angle A \cong \angle D \text{ and } \angle B \cong \angle E$$

Then ...

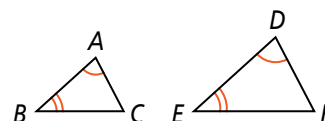
$$\angle C \cong \angle F$$



### Proof of Theorem 4-1: Third Angles Theorem

**Given:**  $\angle A \cong \angle D, \angle B \cong \angle E$

**Prove:**  $\angle C \cong \angle F$



Statements	Reasons
1) $\angle A \cong \angle D, \angle B \cong \angle E$	1) Given
2) $m\angle A = m\angle D, m\angle B = m\angle E$	2) Def. of $\cong$
3) $m\angle A + m\angle B + m\angle C = 180,$ $m\angle D + m\angle E + m\angle F = 180$	3) $\triangle$ Angle-Sum Thm.
4) $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	4) Subst. Prop.
5) $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	5) Subst. Prop.
6) $m\angle C = m\angle F$	6) Subtraction Prop. of =
7) $\angle C \cong \angle F$	7) Def. of $\cong$

## Plan

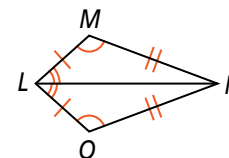
You know four pairs of congruent parts. What else do you need to prove the triangles congruent? You need a third pair of congruent sides and a third pair of congruent angles.



### Problem 4 Proving Triangles Congruent

**Given:**  $\overline{LM} \cong \overline{LO}$ ,  $\overline{MN} \cong \overline{ON}$ ,  
 $\angle M \cong \angle O$ ,  $\angle MLN \cong \angle OLN$

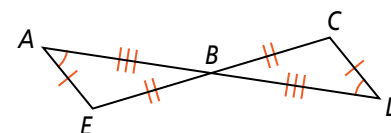
**Prove:**  $\triangle LMN \cong \triangle LON$



Statements	Reasons
1) $\overline{LM} \cong \overline{LO}$ , $\overline{MN} \cong \overline{ON}$	1) Given
2) $\overline{LN} \cong \overline{LN}$	2) Reflexive Property of $\cong$
3) $\angle M \cong \angle O$ , $\angle MLN \cong \angle OLN$	3) Given
4) $\angle MNL \cong \angle ONL$	4) Third Angles Theorem
5) $\triangle LMN \cong \triangle LON$	5) Definition of $\cong$ triangles



**Got It?** 4. **Given:**  $\angle A \cong \angle D$ ,  $\overline{AE} \cong \overline{DC}$ ,  
 $\overline{EB} \cong \overline{CB}$ ,  $\overline{BA} \cong \overline{BD}$   
**Prove:**  $\triangle AEB \cong \triangle DCB$



## Lesson Check

### Do you know HOW?

Complete the following statements.

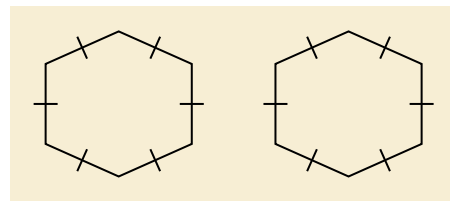
- Given:**  $\triangle QXR \cong \triangle NYC$ 
  - $\overline{QX} \cong$  ?
  - $\angle Y \cong$  ?
- Given:**  $\triangle BAT \cong \triangle FOR$ 
  - $\overline{TA} \cong$  ?
  - $\angle R \cong$  ?
- Given:**  $\triangle BAND \cong \triangle LUCK$ 
  - $\angle U \cong$  ?
  - $\overline{DB} \cong$  ?
  - $\triangle NDBA \cong$  ?
- In  $\triangle MAP$  and  $\triangle TIE$ ,  $\angle A \cong \angle I$  and  $\angle P \cong \angle E$ .
  - What is the relationship between  $\angle M$  and  $\angle T$ ?
  - If  $m\angle A = 52$  and  $m\angle P = 36$ , what is  $m\angle T$ ?

### Do you UNDERSTAND?



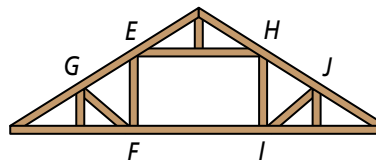
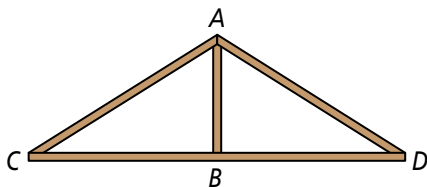
**MATHEMATICAL PRACTICES**

- Open-Ended** When do you think you might need to know that things are congruent in your everyday life?
- If each angle in one triangle is congruent to its corresponding angle in another triangle, are the two triangles congruent? Explain.
- Error Analysis** Walter sketched the diagram below. He claims it shows that the two polygons are congruent. What information is missing to support his claim?



## A Practice

8. **Construction** Builders use the king post truss (below left) for the top of a simple structure. In this truss,  $\triangle ABC \cong \triangle ABD$ . List the congruent corresponding parts. ← See Problem 1.



9. The attic frame truss (above right) provides open space in the center for storage. In this truss,  $\triangle EFG \cong \triangle HIJ$ . List the congruent corresponding parts.

$\triangle LMC \cong \triangle BJK$ . Complete the congruence statements.

10.  $\overline{LC} \cong \underline{\hspace{1cm}} ?$

11.  $\overline{KJ} \cong \underline{\hspace{1cm}} ?$

12.  $\overline{JB} \cong \underline{\hspace{1cm}} ?$

13.  $\angle L \cong \underline{\hspace{1cm}} ?$

14.  $\angle K \cong \underline{\hspace{1cm}} ?$

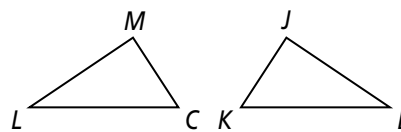
15.  $\angle M \cong \underline{\hspace{1cm}} ?$

16.  $\triangle CML \cong \underline{\hspace{1cm}} ?$

17.  $\triangle KBJ \cong \underline{\hspace{1cm}} ?$

18.  $\triangle MLC \cong \underline{\hspace{1cm}} ?$

19.  $\triangle JKB \cong \underline{\hspace{1cm}} ?$



$POLY \cong SIDE$ . List each of the following.

20. four pairs of congruent sides

21. four pairs of congruent angles

At an archeological site, the remains of two ancient step pyramids are congruent. If  $ABCD \cong EFGH$ , find each of the following. (Diagrams are not to scale.) ← See Problem 2.

22.  $AD$

23.  $GH$

24.  $m\angle GHE$

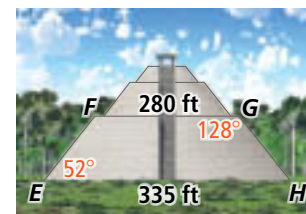
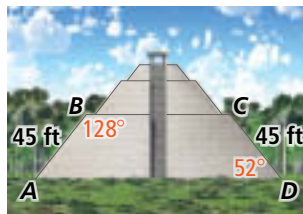
25.  $m\angle BAD$

26.  $EF$

27.  $BC$

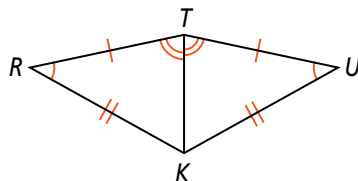
28.  $m\angle DCB$

29.  $m\angle EFG$

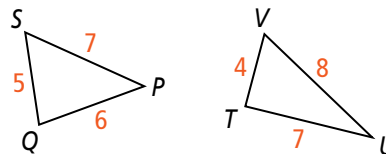


For Exercises 30 and 31, can you conclude that the triangles are congruent? Justify your answers. ← See Problem 3.

30.  $\triangle TRK$  and  $\triangle TUK$



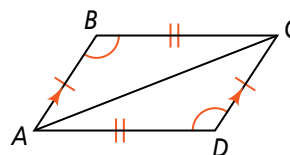
31.  $\triangle SPQ$  and  $\triangle TUV$



**B Apply**

**32. Given:**  $\overline{AB} \parallel \overline{DC}$ ,  $\angle B \cong \angle D$ ,  
**Proof**  $\overline{AB} \cong \overline{DC}$ ,  $\overline{BC} \cong \overline{AD}$

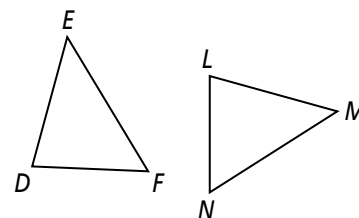
**Prove:**  $\triangle ABC \cong \triangle CDA$



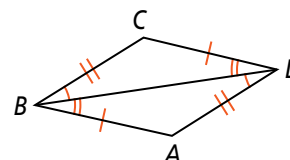
◀ See Problem 4.

**33.** If  $\triangle DEF \cong \triangle LMN$ , which of the following must be a correct congruence statement?

- (A)  $\overline{DE} \cong \overline{LN}$       (C)  $\angle N \cong \angle F$   
 (B)  $\overline{FE} \cong \overline{NL}$       (D)  $\angle M \cong \angle F$



**34. Reasoning** Randall says he can use the information in the figure to prove  $\triangle BCD \cong \triangle DAB$ . Is he correct? Explain.

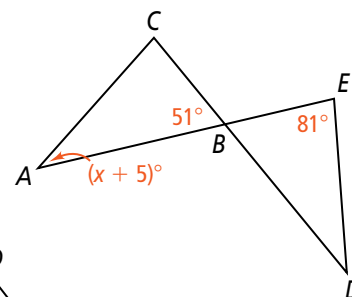


**Algebra**  $\triangle ABC \cong \triangle DEF$ . Find the measures of the given angles or the lengths of the given sides.

- 35.**  $m\angle A = x + 10$ ,  $m\angle D = 2x$       **36.**  $m\angle B = 3y$ ,  $m\angle E = 6y - 12$   
**37.**  $BC = 3z + 2$ ,  $EF = z + 6$       **38.**  $AC = 7a + 5$ ,  $DF = 5a + 9$

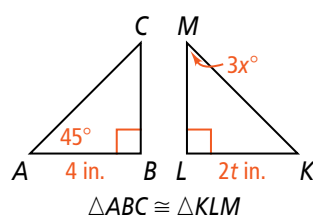
**39. Think About a Plan**  $\triangle ABC \cong \triangle DBE$ . Find the value of  $x$ .

- What does it mean for two triangles to be congruent?
- Which angle measures do you already know?
- How can you find the missing angle measure in a triangle?

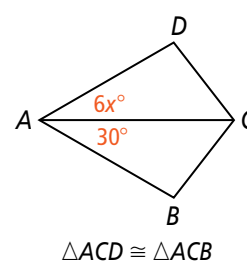


**Algebra** Find the values of the variables.

**40.**

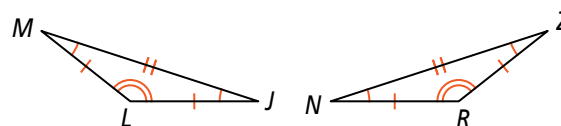


**41.**

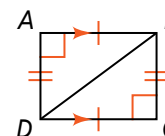


**42.** Complete in two different ways:  
 $\triangle JLM \cong \underline{\hspace{1cm}}$ .

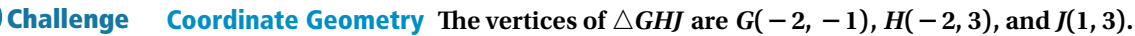
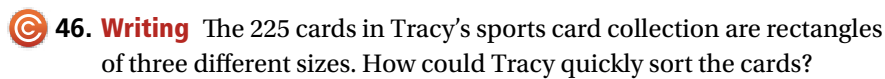
**43. Open-Ended** Write a congruence statement for two triangles. List the congruent sides and angles.



**44. Given:**  $\overline{AB} \perp \overline{AD}$ ,  $\overline{BC} \perp \overline{CD}$ ,  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AD} \cong \overline{CB}$ ,  $\overline{AB} \parallel \overline{CD}$   
**Proof** **Prove:**  $\triangle ABD \cong \triangle CDB$



**Prove:**  $\triangle PRS \cong \triangle QTS$



**48.** If  $L$  and  $M$  have coordinates  $L(3, -3)$  and  $M(6, -3)$ , how many pairs of coordinates are possible for  $K$ ? Find one such pair.

**49. a.** How many quadrilaterals (convex and concave) with different shapes or sizes can you make on a three-by-three geoboard? Sketch them. One is shown at the right.

**b.** How many quadrilaterals of each type are there?



## MP 1

- Copy and label the diagram. Include all the given information in your diagram.
- Which angles do you know to be congruent? Explain.
- Which sides do you know to be congruent? Explain.
- Can you conclude that  $\triangle ABC \cong \triangle AYX$  using the definition of congruent triangles? If not, what additional information would you need?